H

National Qualifications 2018

X747/76/11

Mathematics Paper 1 (Non-Calculator)

THURSDAY, 3 MAY

9:00 AM – 10:10 AM

Total marks — 60

Attempt ALL questions.

You may NOT use a calculator.

Full credit will be given only to solutions which contain appropriate working.

State the units for your answer where appropriate.

Answers obtained by readings from scale drawings will not receive any credit.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer should not be taken as an indication of how much to write. It is not necessary to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space **you must clearly identify the question number** you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

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FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\mathbf{a.b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

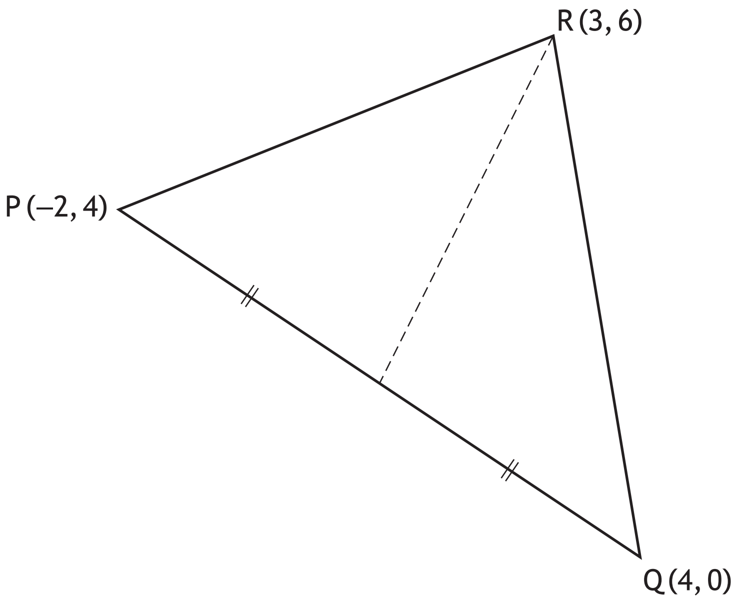
$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

Attempt ALL questions

Total marks — 60

MARKS

1. PQR is a triangle with vertices P (–2, 4), Q (4, 0) and R (3, 6).



Find the equation of the median through R. **3**

2. A function $g(x)$ is defined on $\mathbb{R}$, the set of real numbers, by

$$g(x) = \frac{1}{5}x - 4.$$

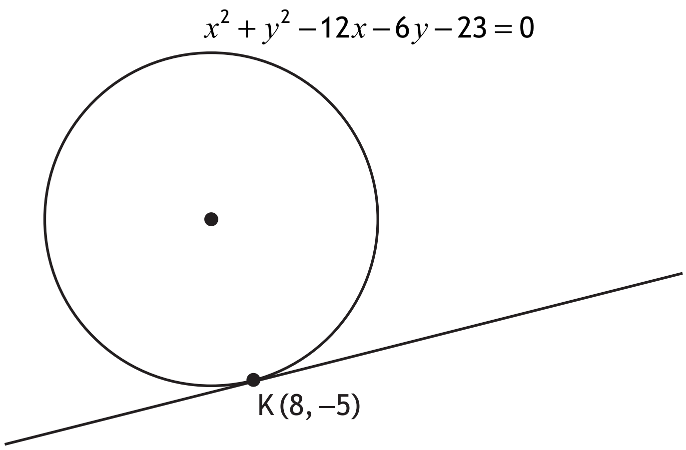
Find the inverse function, $g^{-1}(x)$. **3**

3. Given $h(x) = 3\cos 2x$, find the value of $h'\left(\frac{\pi}{6}\right)$. **3**

[Turn over

MARKS

4. The point K (8, –5) lies on the circle with equation $x^2 + y^2 - 12x - 6y - 23 = 0$.



Find the equation of the tangent to the circle at K. **4**

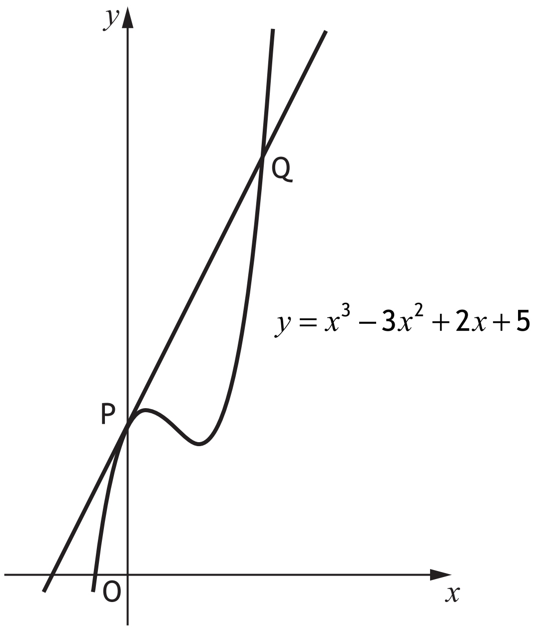
5. A (–3, 4, –7), B (5, t, 5) and C (7, 9, 8) are collinear.

(a) State the ratio in which B divides AC. **1**

(b) State the value of t. **1**

6. Find the value of $\log_5 250 - \frac{1}{3}\log_5 8$. **3**

7. The curve with equation $y = x^3 - 3x^2 + 2x + 5$ is shown on the diagram.



(a) Write down the coordinates of P, the point where the curve crosses the y-axis . **1**

(b) Determine the equation of the tangent to the curve at P. **3**

(c) Find the coordinates of Q, the point where this tangent meets the curve again. **4**

8. A line has equation $y - \sqrt{3}\,x + 5 = 0$.

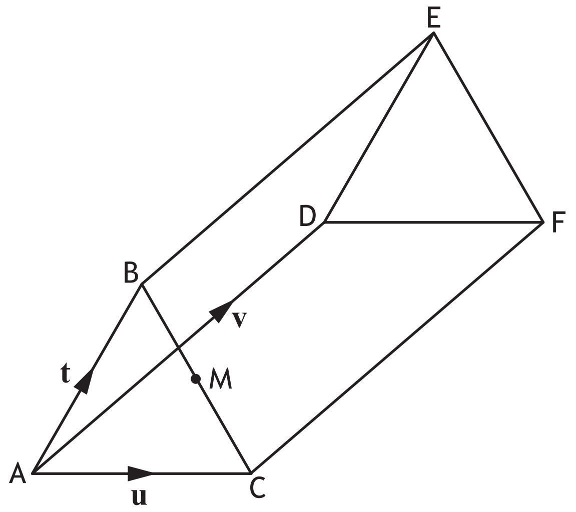
Determine the angle this line makes with the positive direction of the x-axis. **2**

[Turn over

MARKS

9. The diagram shows a triangular prism ABC,DEF.

$\overrightarrow{AB} = \mathbf{t}$, $\overrightarrow{AC} = \mathbf{u}$ and $\overrightarrow{AD} = \mathbf{v}$.



(a) Express $\overrightarrow{BC}$ in terms of $\mathbf{u}$ and $\mathbf{t}$. **1**

M is the midpoint of BC.

(b) Express $\overrightarrow{MD}$ in terms of $\mathbf{t}$, $\mathbf{u}$ and $\mathbf{v}$. **2**

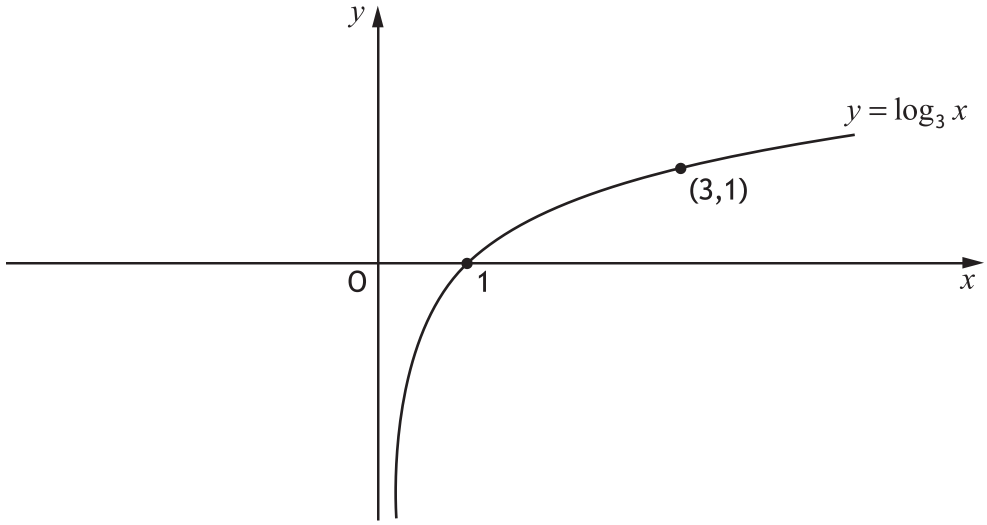
10. Given that

- $\frac{dy}{dx} = 6x^2 - 3x + 4$, and
- $y = 14$ when $x = 2$,

express y in terms of x. **4**

MARKS

11. The diagram shows the curve with equation $y = \log_3 x$.



(a) On the diagram in your answer booklet, sketch the curve with equation $y = 1 - \log_3 x$. **2**

(b) Determine the exact value of the x-coordinate of the point of intersection of the two curves. **3**

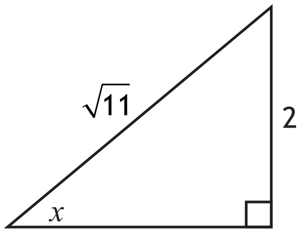
12. Vectors $\mathbf{a}$ and $\mathbf{b}$ are such that $\mathbf{a} = 4\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = -2\mathbf{i} + \mathbf{j} + p\mathbf{k}$.

(a) Express $2\mathbf{a} + \mathbf{b}$ in component form. **1**

(b) Hence find the values of p for which $|2\mathbf{a} + \mathbf{b}| = 7$. **3**

[Turn over for next question

13. The right-angled triangle in the diagram is such that $\sin x = \frac{2}{\sqrt{11}}$ and $0 < x < \frac{\pi}{4}$.



(a) Find the exact value of:

(i) $\sin 2x$ **3**

(ii) $\cos 2x$. **1**

(b) By expressing $\sin 3x$ as $\sin(2x + x)$, find the exact value of $\sin 3x$. **3**

14. Evaluate $\displaystyle\int_{-4}^{9} \frac{1}{\sqrt[3]{(2x+9)^2}}\,dx$. **5**

15. A cubic function, f, is defined on the set of real numbers.

- $(x+4)$ is a factor of $f(x)$
- $x = 2$ is a repeated root of $f(x)$
- $f'(-2) = 0$
- $f'(x) > 0$ where the graph with equation $y = f(x)$ crosses the y-axis

Sketch a possible graph of $y = f(x)$ on the diagram in your answer booklet. **4**

[END OF QUESTION PAPER]

H

National
Qualifications
2018

X747/76/12

Mathematics
Paper 2

THURSDAY, 3 MAY

10:30 AM – 12:00 NOON

Total marks — 70

Attempt ALL questions.

You may use a calculator.

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A/PB

FORMULAE LIST

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Trigonometric formulae:

$$\sin(\mathrm{A} \pm \mathrm{B}) = \sin \mathrm{A} \cos \mathrm{B} \pm \cos \mathrm{A} \sin \mathrm{B}$$

$$\cos(\mathrm{A} \pm \mathrm{B}) = \cos \mathrm{A} \cos \mathrm{B} \mp \sin \mathrm{A} \sin \mathrm{B}$$

$$\sin 2\mathrm{A} = 2 \sin \mathrm{A} \cos \mathrm{A}$$

$$\cos 2\mathrm{A} = \cos^2 \mathrm{A} - \sin^2 \mathrm{A}$$

$$= 2\cos^2 \mathrm{A} - 1$$

$$= 1 - 2\sin^2 \mathrm{A}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
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Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
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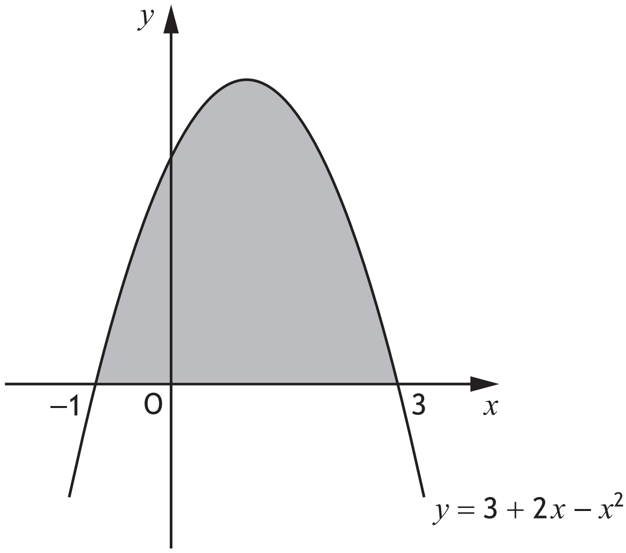
MARKS

Attempt ALL questions

Total marks — 70

1. The diagram shows the curve with equation $y = 3 + 2x - x^2$.



Calculate the shaded area. **4**

2. Vectors $\mathbf{u}$ and $\mathbf{v}$ are defined by $\mathbf{u} = \begin{pmatrix} -1 \\ 4 \\ -3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -7 \\ 8 \\ 5 \end{pmatrix}$.

(a) Find $\mathbf{u}.\mathbf{v}$. **1**

(b) Calculate the acute angle between $\mathbf{u}$ and $\mathbf{v}$. **4**

3. A function, f, is defined on the set of real numbers by $f(x) = x^3 - 7x - 6$.

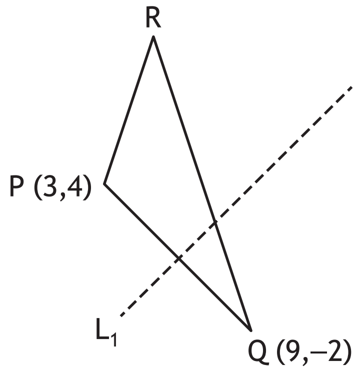
Determine whether f is increasing or decreasing when $x = 2$. **3**

4. Express $-3x^2 - 6x + 7$ in the form $a(x+b)^2 + c$. **3**

[Turn over

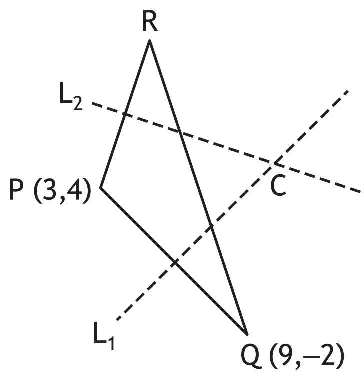
MARKS

5. PQR is a triangle with P(3,4) and Q(9,–2).



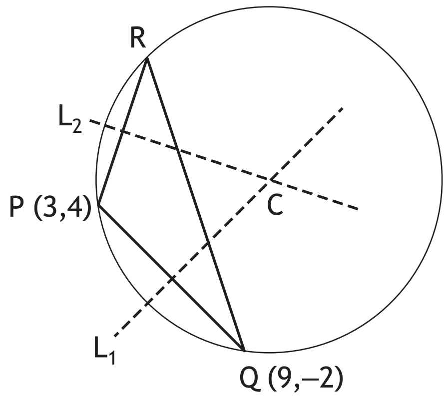
(a) Find the equation of L_1, the perpendicular bisector of PQ. **3**

The equation of L_2, the perpendicular bisector of PR is $3y + x = 25$.



(b) Calculate the coordinates of C, the point of intersection of L_1 and L_2. **2**

C is the centre of the circle which passes through the vertices of triangle PQR.



(c) Determine the equation of this circle. **2**

MARKS

6. Functions, f and g, are given by $f(x) = 3 + \cos x$ and $g(x) = 2x$, $x \in \mathbb{R}$.

(a) Find expressions for

(i) $f(g(x))$ and **2**

(ii) $g(f(x))$. **1**

(b) Determine the value(s) of x for which $f(g(x)) = g(f(x))$ where $0 \leq x < 2\pi$. **6**

7. (a) (i) Show that $(x-2)$ is a factor of $2x^3 - 3x^2 - 3x + 2$. **2**

(ii) Hence, factorise $2x^3 - 3x^2 - 3x + 2$ fully. **2**

The fifth term, u_5, of a sequence is $u_5 = 2a - 3$.

The terms of the sequence satisfy the recurrence relation $u_{n+1} = au_n - 1$.

(b) Show that $u_7 = 2a^3 - 3a^2 - a - 1$. **1**

For this sequence, it is known that

- $u_7 = u_5$
- a limit exists.

(c) (i) Determine the value of a. **3**

(ii) Calculate the limit. **1**

[Turn over

8. (a) Express $2\cos x° - \sin x°$ in the form $k\cos(x-a)°$, $k>0$, $0<a<360$. **4**

(b) Hence, or otherwise, find

(i) the minimum value of $6\cos x° - 3\sin x°$ and **1**

(ii) the value of x for which it occurs where $0 \le x < 360$. **2**

9. A sector with a particular fixed area has radius x cm.

The perimeter, P cm, of the sector is given by

$$P = 2x + \frac{128}{x}.$$

Find the minimum value of P. **6**

10. The equation $x^2 + (m-3)x + m = 0$ has two real and distinct roots.

Determine the range of values for m. **4**

11. A supermarket has been investigating how long customers have to wait at the checkout.

During any half hour period, the percentage, P%, of customers who wait for less than t minutes, can be modelled by

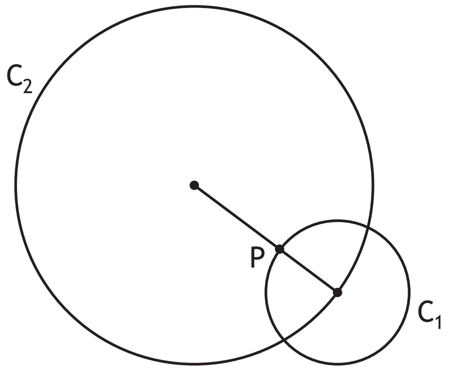
$P = 100\left(1 - e^{kt}\right)$, where k is a constant.

(a) If 50% of customers wait for less than 3 minutes, determine the value of k. **4**

(b) Calculate the percentage of customers who wait for 5 minutes or longer. **2**

12. Circle C_1 has equation $(x-13)^2+(y+4)^2=100$.

Circle C_2 has equation $x^2+y^2+14x-22y+c=0$.



(a) (i) Write down the coordinates of the centre of C_1. **1**

(ii) The centre of C_1 lies on the circumference of C_2.

Show that $c=-455$. **1**

The line joining the centres of the circles intersects C_1 at P.

(b) (i) Determine the ratio in which P divides the line joining the centres of the circles. **2**

(ii) Hence, or otherwise, determine the coordinates of P. **2**

P is the centre of a third circle, C_3.

C_2 touches C_3 internally.

(c) Determine the equation of C_3. **1**

[END OF QUESTION PAPER]