H

National
Qualifications
2017

X747/76/11

Mathematics
Paper 1
(Non-Calculator)

FRIDAY, 5 MAY

9:00 AM – 10:10 AM

Total marks — 60

Attempt ALL questions.

You may NOT use a calculator.

Full credit will be given only to solutions which contain appropriate working.

State the units for your answer where appropriate.

Answers obtained by readings from scale drawings will not receive any credit.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer should not be taken as an indication of how much to write. It is not necessary to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space **you must clearly identify the question number** you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

X7477611

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\mathbf{a.b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2\sin A \cos A$$

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

ATTEMPT ALL QUESTIONS **MARKS**

Total marks — 60

1. Functions f and g are defined on suitable domains by $f(x) = 5x$ and $g(x) = 2\cos x$.

(a) Evaluate $f(g(0))$. **1**

(b) Find an expression for $g(f(x))$. **2**

2. The point P (–2, 1) lies on the circle $x^2 + y^2 - 8x - 6y - 15 = 0$.

Find the equation of the tangent to the circle at P. **4**

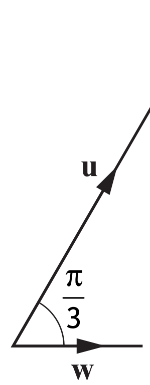
3. Given $y = (4x-1)^{12}$, find $\frac{dy}{dx}$. **2**

4. Find the value of k for which the equation $x^2 + 4x + (k-5) = 0$ has equal roots. **3**

5. Vectors $\mathbf{u}$ and $\mathbf{v}$ are $\begin{pmatrix} 5 \\ 1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -8 \\ 6 \end{pmatrix}$ respectively.

(a) Evaluate $\mathbf{u}.\mathbf{v}$. **1**

(b)



Vector $\mathbf{w}$ makes an angle of $\frac{\pi}{3}$ with $\mathbf{u}$ and $|\mathbf{w}| = \sqrt{3}$.

Calculate $\mathbf{u}.\mathbf{w}$. **3**

6. A function, h, is defined by $h(x) = x^3 + 7$, where $x \in \mathbb{R}$.

Determine an expression for $h^{-1}(x)$. **3**

7. A $(-3, 5)$, B $(7, 9)$ and C $(2, 11)$ are the vertices of a triangle.

Find the equation of the median through C. **3**

8. Calculate the rate of change of $d(t) = \frac{1}{2t}$, $t \neq 0$, when $t = 5$. **3**

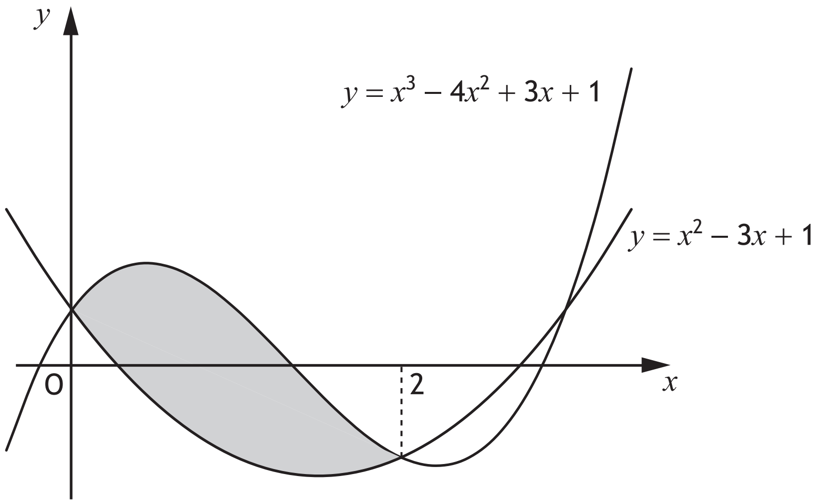
9. A sequence is generated by the recurrence relation $u_{n+1} = m u_n + 6$ where m is a constant.

(a) Given $u_1 = 28$ and $u_2 = 13$, find the value of m. **2**

(b) (i) Explain why this sequence approaches a limit as $n \to \infty$. **1**

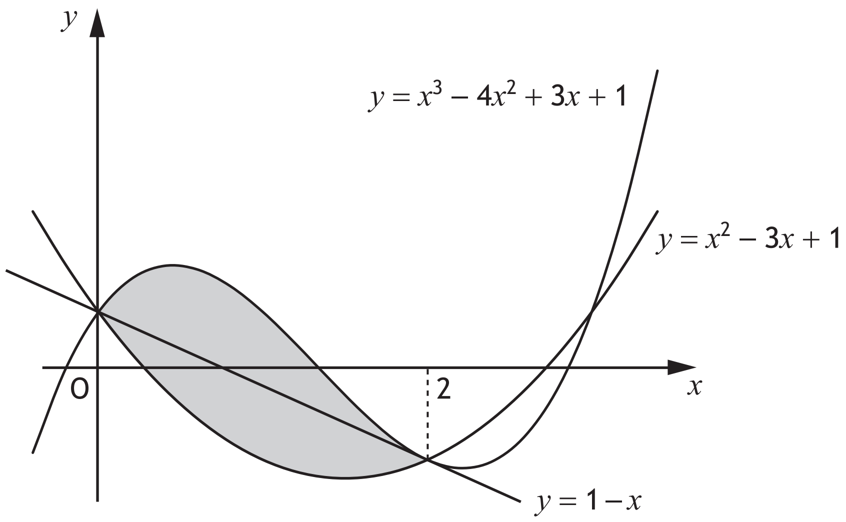
(ii) Calculate this limit. **2**

10. Two curves with equations $y = x^3 - 4x^2 + 3x + 1$ and $y = x^2 - 3x + 1$ intersect as shown in the diagram.



(a) Calculate the shaded area. **5**

The line passing through the points of intersection of the curves has equation $y = 1 - x$.



(b) Determine the fraction of the shaded area which lies below the line $y = 1 - x$. **4**

[Turn over

MARKS

11. A and B are the points (–7, 2) and (5, a).

AB is parallel to the line with equation $3y - 2x = 4$.

Determine the value of a. **3**

12. Given that $\log_a 36 - \log_a 4 = \frac{1}{2}$, find the value of a. **3**

13. Find $\int \frac{1}{(5-4x)^{\frac{1}{2}}}\,dx,\ \ x < \frac{5}{4}$. **4**

14. (a) Express $\sqrt{3}\sin x° - \cos x°$ in the form $k\sin(x-a)°$, where $k > 0$ and $0 < a < 360$. **4**

(b) Hence, or otherwise, sketch the graph with equation $y = \sqrt{3}\sin x° - \cos x°$, $0 \le x \le 360$. **3**

Use the diagram provided in the answer booklet.

15. A quadratic function, f, is defined on $\mathbb{R}$, the set of real numbers.

Diagram 1 shows part of the graph with equation $y = f(x)$.
The turning point is (2, 3).

Diagram 2 shows part of the graph with equation $y = h(x)$.
The turning point is (7, 6).

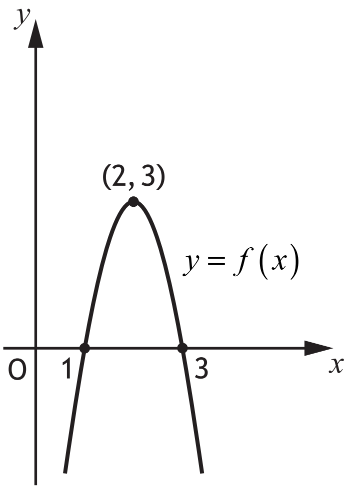


Diagram 1

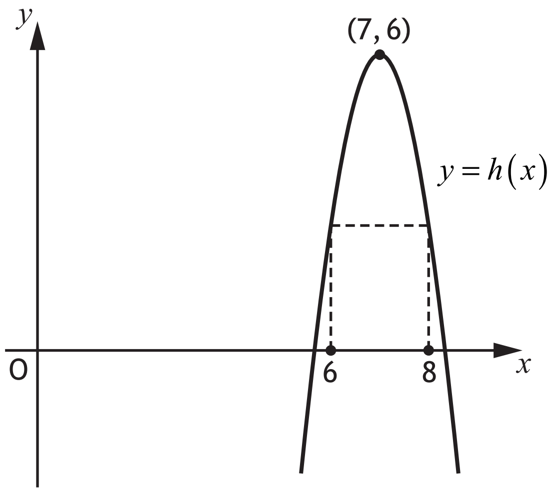


Diagram 2

(a) Given that $h(x) = f(x + a) + b$.

Write down the values of a and b. **2**

(b) It is known that $\int_1^3 f(x)\,dx = 4$.

Determine the value of $\int_6^8 h(x)\,dx$. **1**

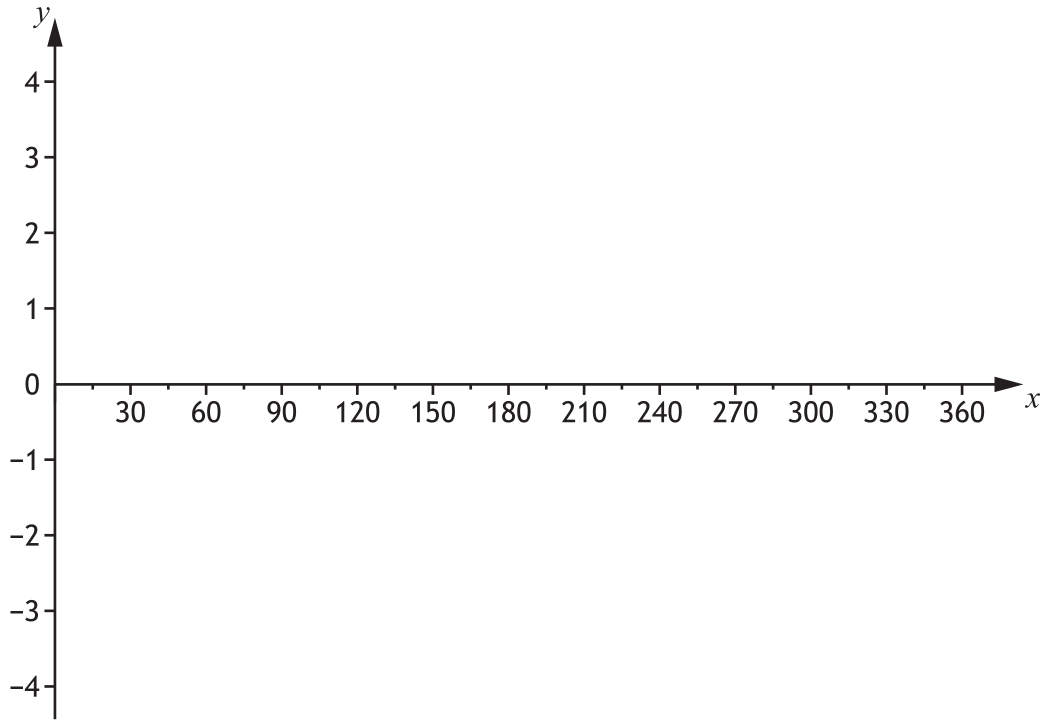
(c) Given $f'(1) = 6$, state the value of $h'(8)$. **1**

[END OF QUESTION PAPER]

QUESTION NUMBER

DO NOT WRITE IN THIS MARGIN

14.(b) **An additional diagram, if required, can be found on Page 13.**



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National
Qualifications
2017

X747/76/12

Mathematics
Paper 2

FRIDAY, 5 MAY

10:30 AM – 12:00 NOON

Total marks — 70

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X7477612

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MARKS

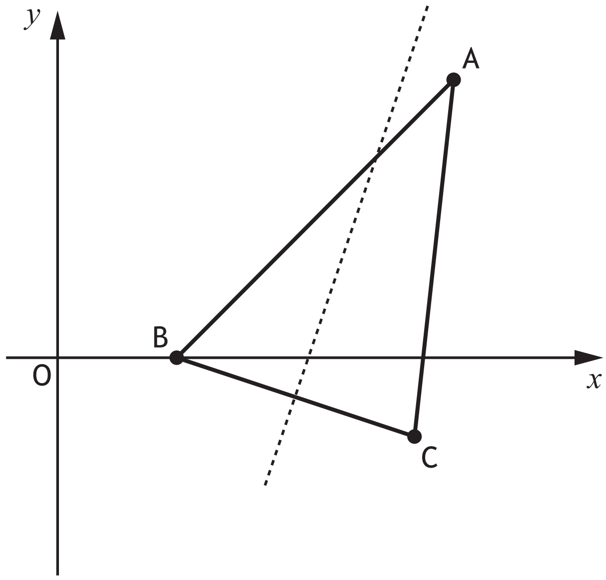
Attempt ALL questions

Total marks — 70

1. Triangle ABC is shown in the diagram below.

   The coordinates of B are (3,0) and the coordinates of C are (9,–2).

   The broken line is the perpendicular bisector of BC.



   (a) Find the equation of the perpendicular bisector of BC. **4**

   (b) The line AB makes an angle of 45° with the positive direction of the x-axis.

   Find the equation of AB. **2**

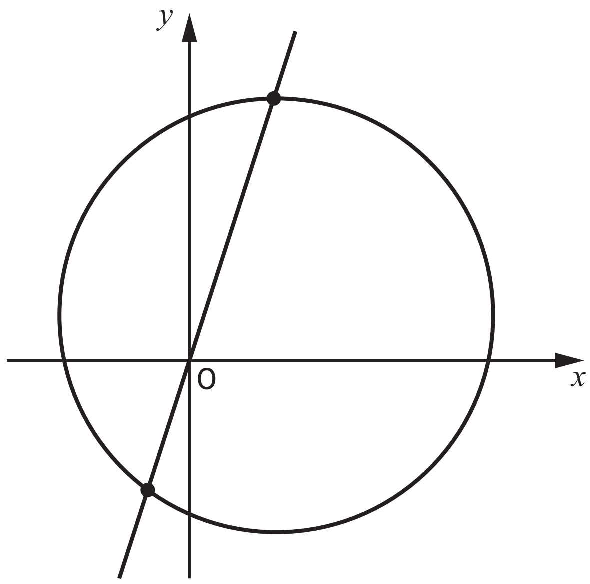
   (c) Find the coordinates of the point of intersection of AB and the perpendicular bisector of BC. **2**

2. (a) Show that $(x-1)$ is a factor of $f(x)=2x^3-5x^2+x+2$. **2**

   (b) Hence, or otherwise, solve $f(x)=0$. **3**

[Turn over

3. The line $y = 3x$ intersects the circle with equation $(x-2)^2 + (y-1)^2 = 25$.



Find the coordinates of the points of intersection. **5**

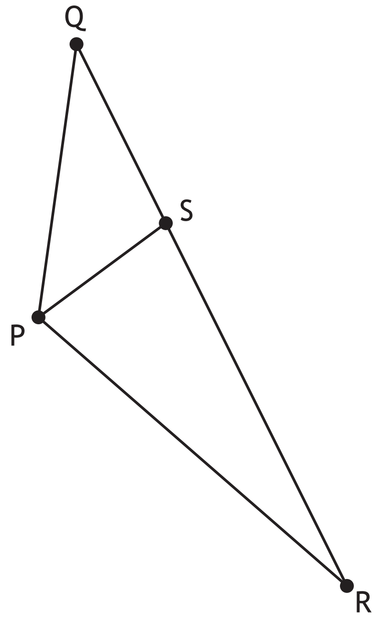
4. (a) Express $3x^2 + 24x + 50$ in the form $a(x+b)^2 + c$. **3**

(b) Given that $f(x) = x^3 + 12x^2 + 50x - 11$, find $f'(x)$. **2**

(c) Hence, or otherwise, explain why the curve with equation $y = f(x)$ is strictly increasing for all values of x. **2**

MARKS

5. In the diagram, $\overrightarrow{PR} = 9\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ and $\overrightarrow{RQ} = -12\mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$.



(a) Express $\overrightarrow{PQ}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. **2**

The point S divides QR in the ratio 1:2.

(b) Show that $\overrightarrow{PS} = \mathbf{i} - \mathbf{j} + 4\mathbf{k}$. **2**

(c) Hence, find the size of angle QPS. **5**

6. Solve $5\sin x - 4 = 2\cos 2x$ for $0 \le x < 2\pi$. **5**

7. (a) Find the x-coordinate of the stationary point on the curve with equation $y = 6x - 2\sqrt{x^3}$. **4**

(b) Hence, determine the greatest and least values of y in the interval $1 \le x \le 9$. **3**

[Turn over

MARKS

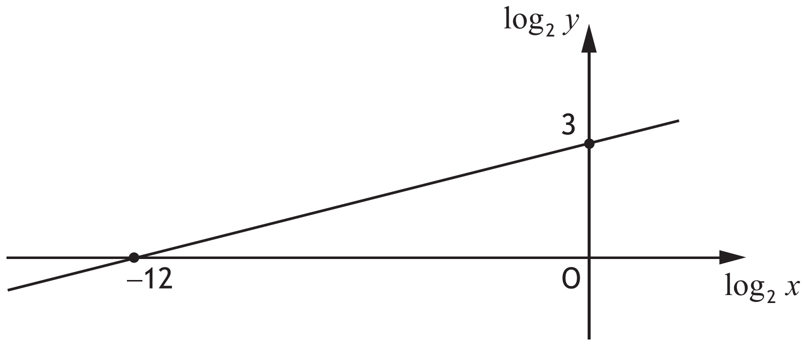
8. Sequences may be generated by recurrence relations of the form $u_{n+1} = k\,u_n - 20$, $u_0 = 5$ where $k \in \mathbb{R}$.

(a) Show that $u_2 = 5k^2 - 20k - 20$. **2**

(b) Determine the range of values of k for which $u_2 < u_0$. **4**

9. Two variables, x and y, are connected by the equation $y = kx^n$.

The graph of $\log_2 y$ against $\log_2 x$ is a straight line as shown.

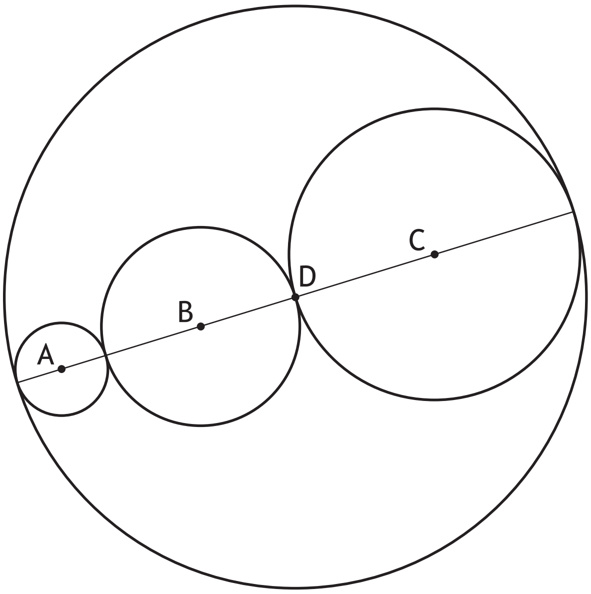


Find the values of k and n. **5**

MARKS

10. (a) Show that the points A(–7, –2), B(2, 1) and C(17, 6) are collinear. **3**

Three circles with centres A, B and C are drawn inside a circle with centre D as shown.



The circles with centres A, B and C have radii r_A, r_B and r_C respectively.

- $r_A = \sqrt{10}$
- $r_B = 2r_A$
- $r_C = r_A + r_B$

(b) Determine the equation of the circle with centre D. **4**

11. (a) Show that $\dfrac{\sin 2x}{2\cos x} - \sin x \cos^2 x = \sin^3 x$, where $0 < x < \dfrac{\pi}{2}$. **3**

(b) Hence, differentiate $\dfrac{\sin 2x}{2\cos x} - \sin x \cos^2 x$, where $0 < x < \dfrac{\pi}{2}$. **3**

[END OF QUESTION PAPER]